A Phenomenology of the Baryon Spectrum from Lattice QCD

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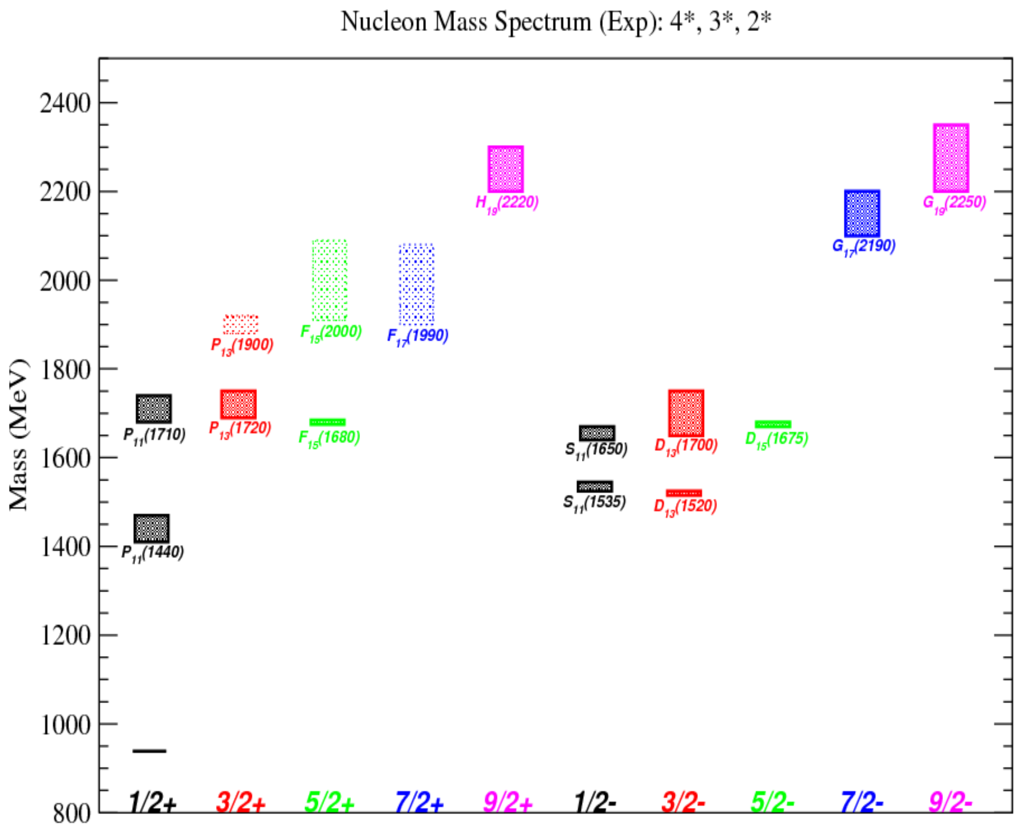
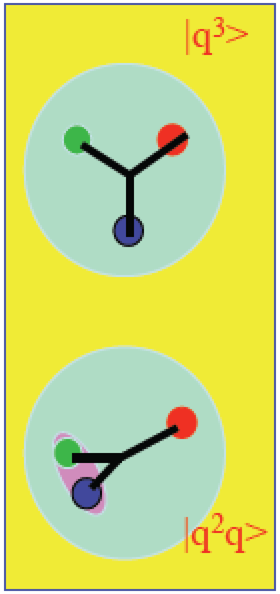
Auspices of the Hadron Spectrum Collaboration

Baryon Spectrum

"Missing resonance problem"

- What are collective modes?
- What is the structure of the states?





Nucleon spectrum

PDG uncertainty on B-W mass

Lattice QCD

Goal: resolve highly excited states

$N_f = 2 + 1$ (u,d + s)

Anisotropic lattices:

$(a_s)^{-1} \sim 1.6$ GeV, $(a_t)^{-1} \sim 5.6$ GeV

0810.3588, 0909.0200, 1004.4930

Spectrum from variational method

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

$$Z_i^{\mathfrak{n}} \equiv \langle \mathfrak{n} |\, \Phi_i \,| 0 \rangle$$

Matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

Diagonalize:
- eigenvalues $\rightarrow$ spectrum
- eigenvectors $\rightarrow$ spectral "overlaps"

Each state optimal combination of Φ_i

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}} \Phi_1 + v_2^{\mathfrak{n}} \Phi_2 + \dots$$

Benefit: orthogonality for near degenerate states

Operator construction

Baryons : permutations of 3 objects

Permutation group S_3: 3 representations

- Symmetric: 1-dimensional
  - e.g., uud+udu+duu
- Antisymmetric: 1-dimensional
  - e.g., uud-udu+duu-…
- Mixed: 2-dimensional
  - e.g., udu - duu & 2duu - udu - uud

Color antisymmetric $\rightarrow$ Require Space [Flavor Spin] symmetric

Classify operators by these permutation symmetries:
- Leads to rich structure

1104.5152

Orbital angular momentum via derivatives

Couple derivatives onto single-site spinors:
Enough D's – build any J,M

$$\mathcal{O}^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}\right]_i \left[\vec{D}\right]_j [\Psi]_k$$

Only using symmetries of continuum QCD

$$\text{Operator}_S \leftarrow \text{Derivatives} \quad \left[\text{Flavor} \quad \text{Dirac}\right]$$

Use all possible **operators** up to 2 derivatives
(transforms like 2 units orbital angular momentum)

1104.5152

Baryon operator basis

3-quark operators with up to two covariant derivatives – projected into definite isospin and continuum J^P

$$\text{Operators} \leftarrow \left(\left[\text{Flavor} \quad \text{Dirac} \right] \quad \text{Space}_{\text{symmetry}} \right)^{J^P}$$

Spatial symmetry classification:

Nucleons: $N\ {}^{2S+1}L_{\pi}\ J^P$

J^P	#ops	E.g., spatial symmetries	
$J=1/2^-$	24	$N\ {}^2P_M\ \frac{1}{2}^-$	$N\ {}^4P_M\ \frac{1}{2}^-$
$J=3/2^-$	28	$N\ {}^2P_M\ 3/2^-$	$N\ {}^4P_M\ 3/2^-$
$J=5/2^-$	16	$N\ {}^4P_M\ 5/2^-$	
$J=1/2^+$	24	$N\ {}^2S_S\ \frac{1}{2}^+$ $N\ {}^2S_M\ \frac{1}{2}^+$	$N\ {}^4D_M\ \frac{1}{2}^+$ $N\ {}^2P_A\ \frac{1}{2}^+$
$J=3/2^+$	28	$N\ {}^2D_S 3/2^+$ $N\ {}^2D_M 3/2^+$ $N\ {}^2P_A\ 3/2^+$	$N\ {}^4S_M 3/2^+$ $N\ {}^4D_M 3/2^+$
$J=5/2^+$	16	$N\ {}^2D_S 5/2^+$ $N\ {}^2D_M 5/2^+$	$N\ {}^4D_M 5/2^+$
$J=7/2^+$	4	$N\ {}^4D_M 7/2^+$	

Symmetry crucial for spectroscopy

By far the largest operator basis ever used for such calculations

Operators are not states

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

Full basis of operators: many operators can create same state

Spectral “overlaps”

$$\langle \mathfrak{n};\, J^P \,|\, \Phi_i \,|\, 0 \rangle \;=\; Z_i^{\mathfrak{n}}$$

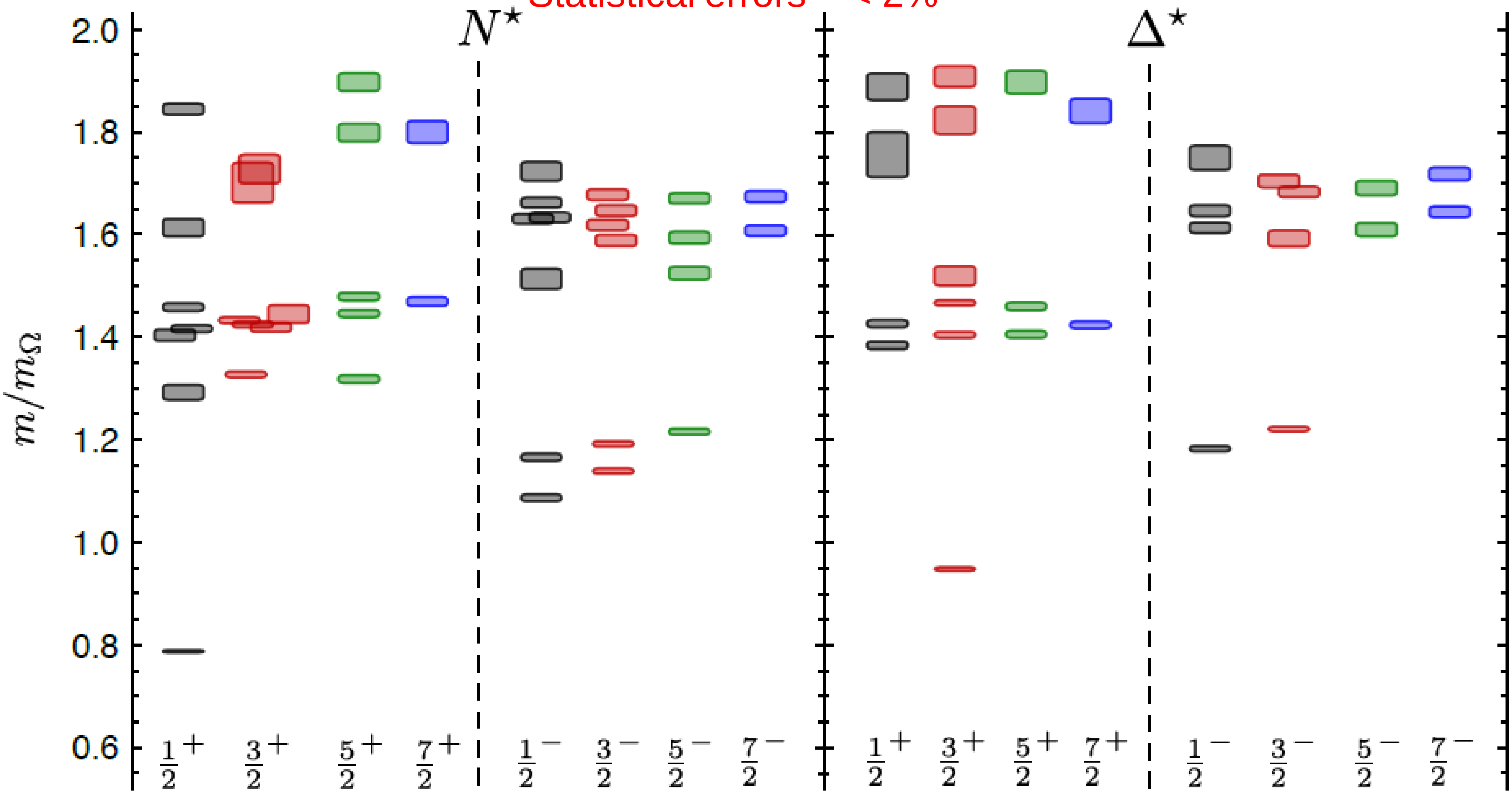
States may have subset of allowed symmetries

Spin identified Nucleon & Delta spectrum

arXiv:1104.5152

$m_\pi \sim 520$MeV

Statistical errors < 2%



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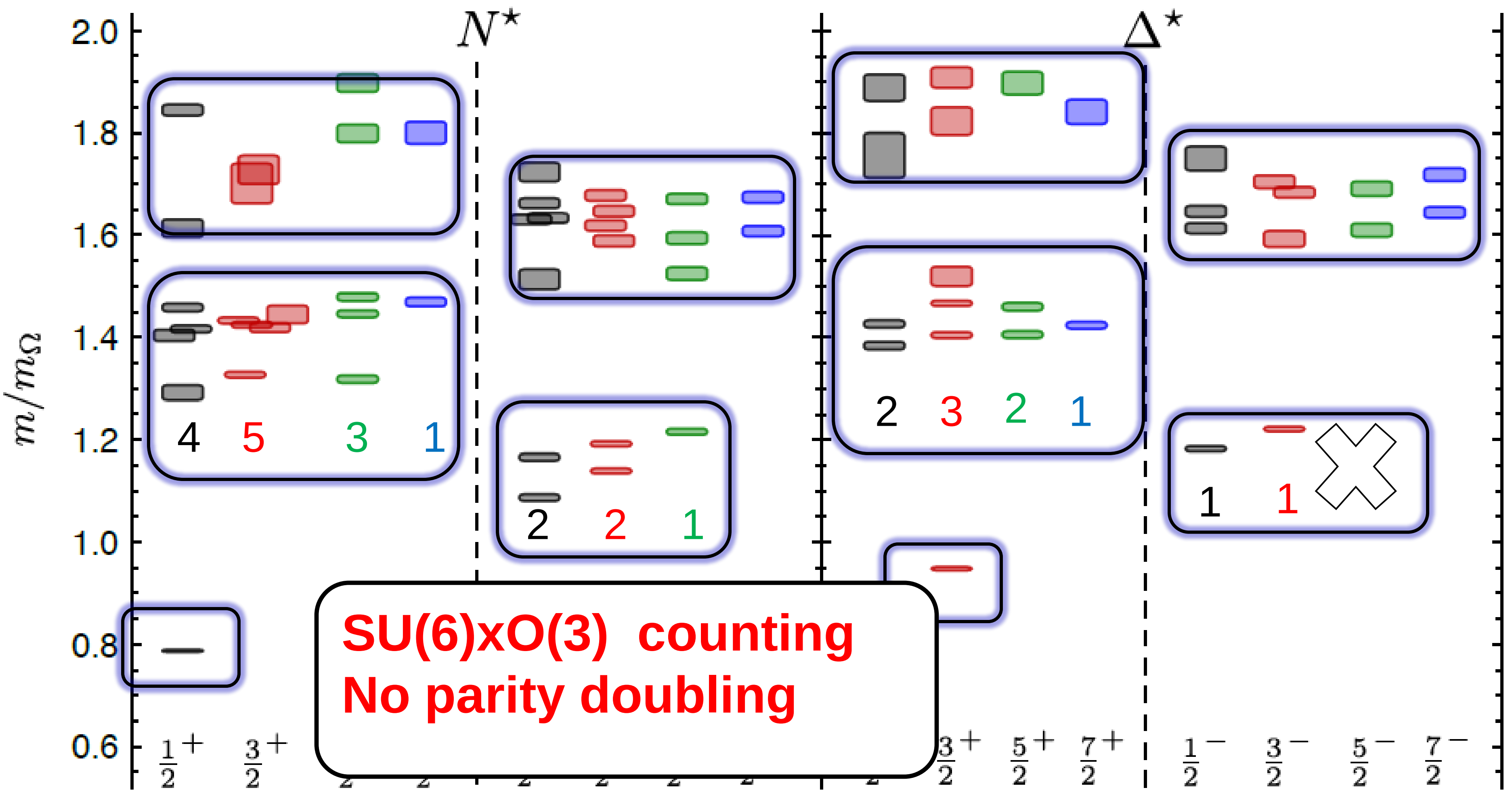


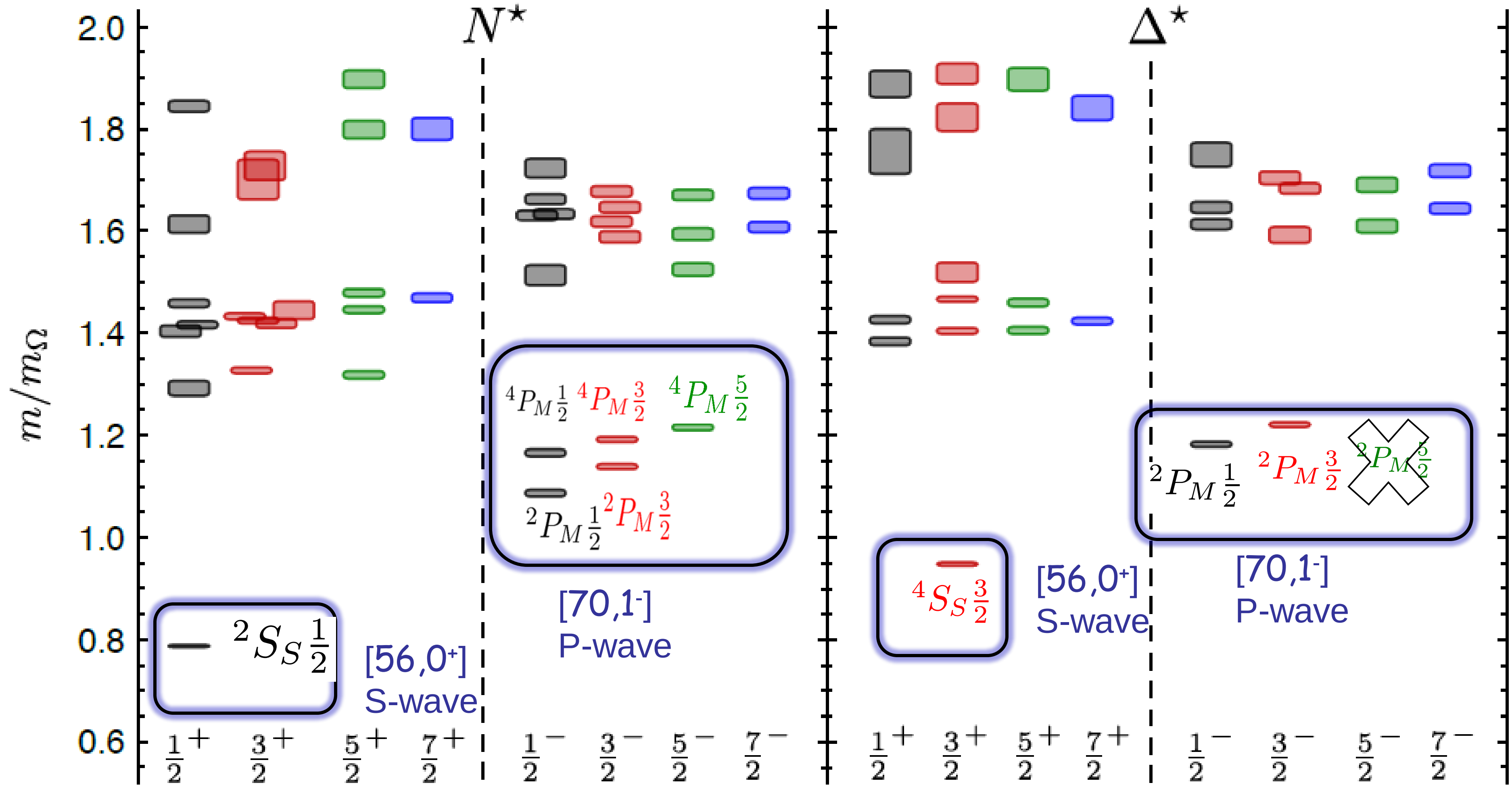
SU(6)xO(3) counting
No parity doubling

Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

arXiv:1104.5152

$m_\pi \sim 520 MeV$



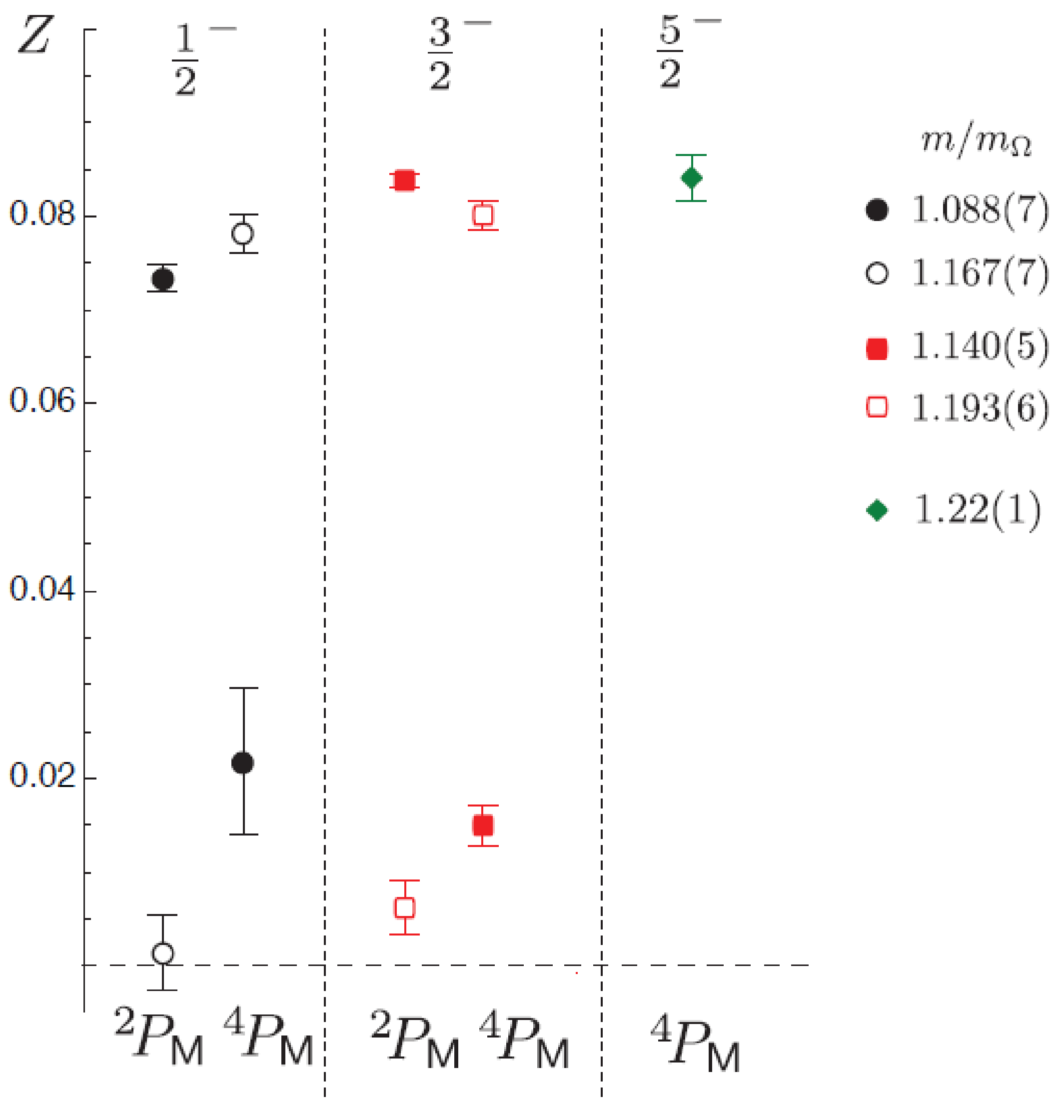
Nucleon J^-

Overlaps

$$Z_i^{\mathfrak{n}} = \langle J^- \mid \Phi_i \mid 0 \rangle$$

Little mixing in each J^-

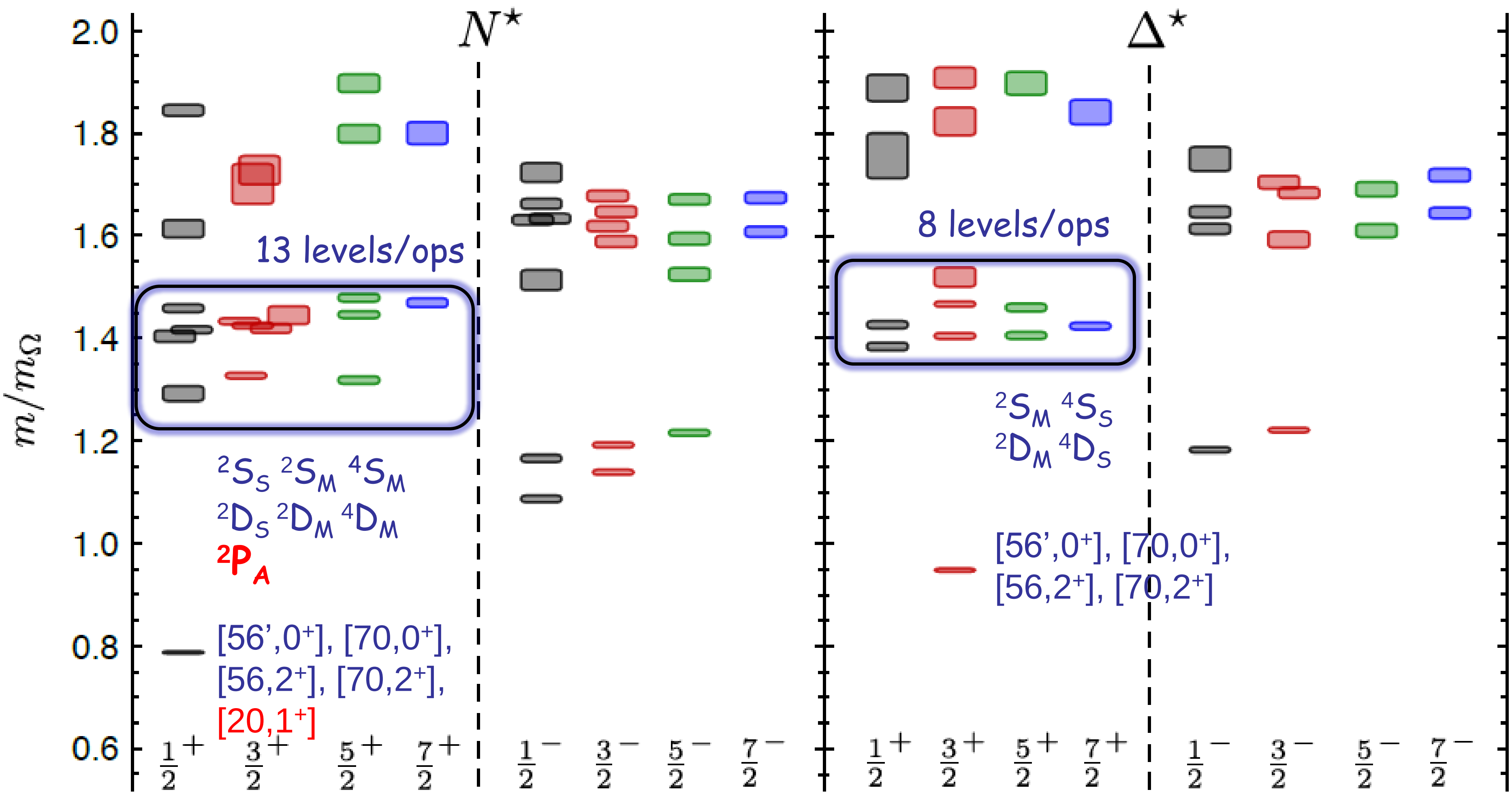
Nearly "pure" [S= 1/2 & 3/2] 1^-



N=2 J$^+$ Nucleon & Delta spectrum

Discern structure: spectral overlaps

Significant mixing in J$^+$



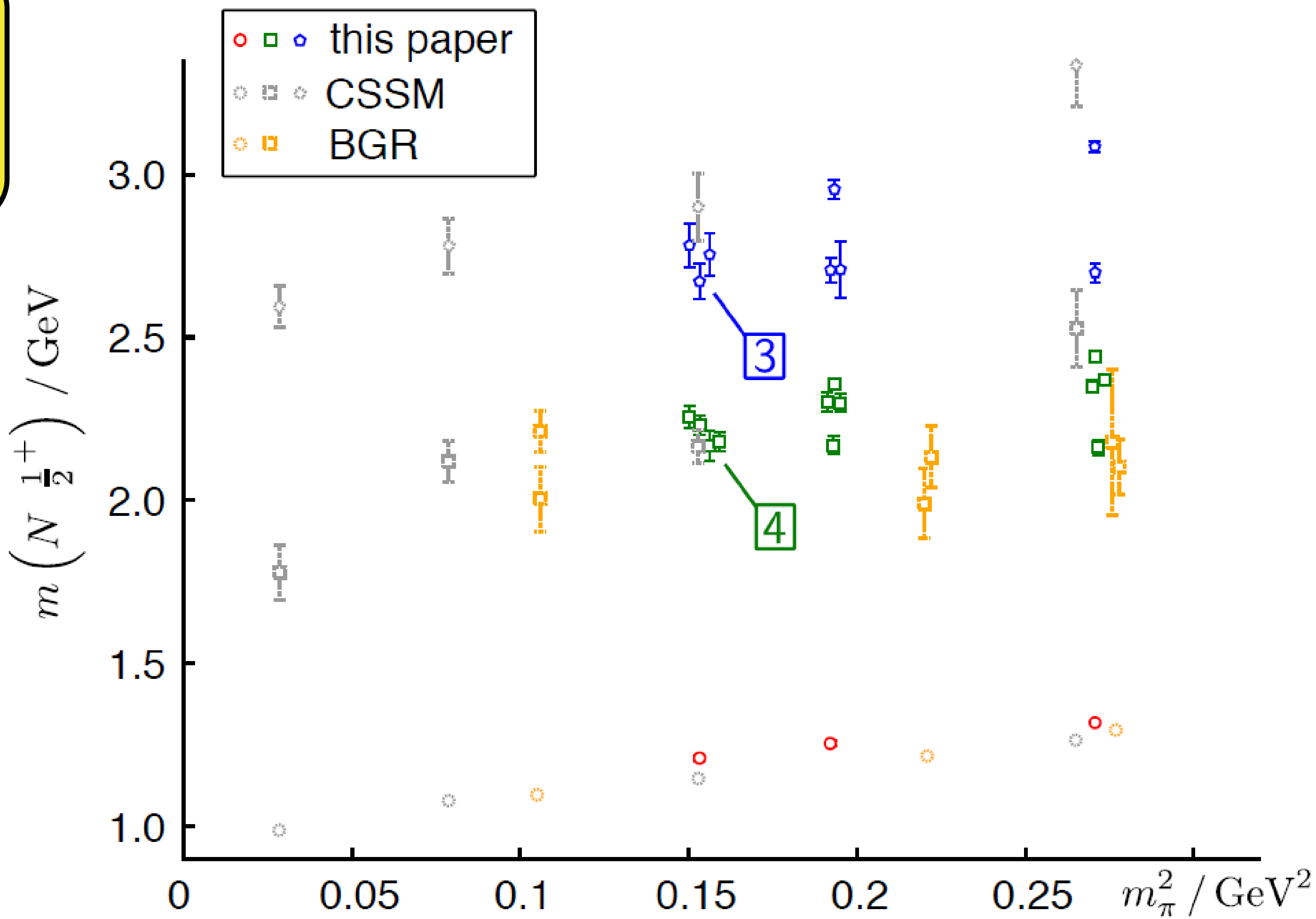
Roper??

Near degeneracy in $\frac{1}{2}^+$ consistent with SU(6) O(3) but heavily mixed

Discrepancies??

Operator basis – spatial structure

What else?
Multi-particle operators



Prospects

- Strong effort in excited state spectroscopy
  - New operator & correlator constructions $\rightarrow$ high lying states
- Results for baryon excited state spectrum:
  - No "freezing" of degrees of freedom nor parity doubling
  - Broadly consistent with non-relativistic quark model
  - Add multi-particles $\rightarrow$ baryon spectrum becomes denser
- Short-term plans: resonance determination!
  - Lighter quark masses
  - Extract couplings in multi-channel systems

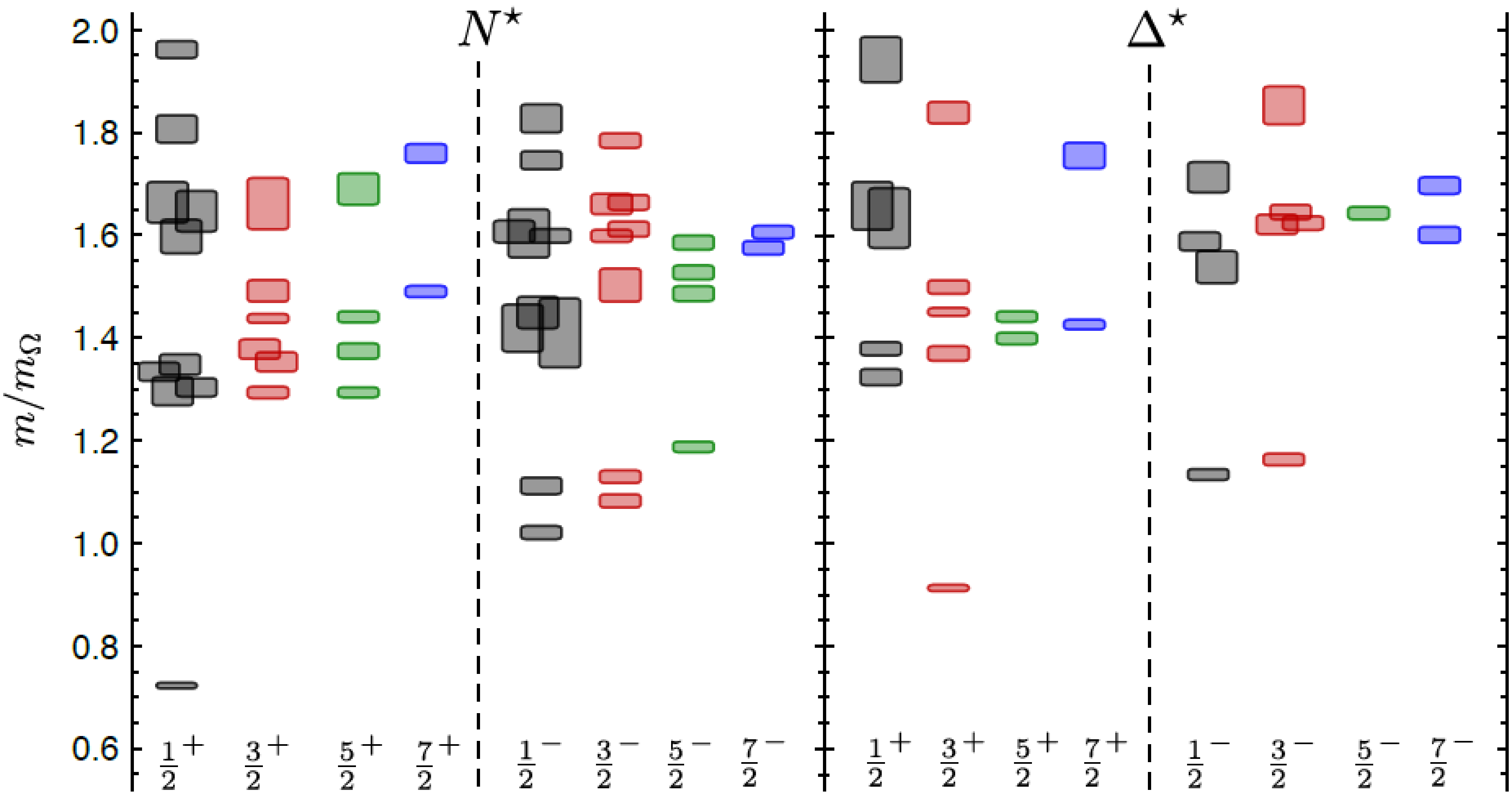
Backup slides

- The end

N & Δ spectrum: lower pion mass

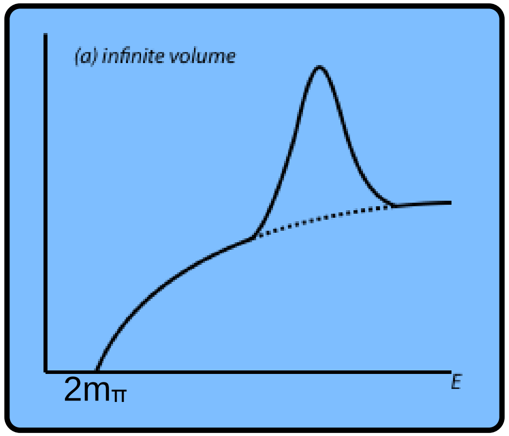
Still bands of states with same counting
More mixing in nucleon N=2 J^+

m_π ~ 400 MeV



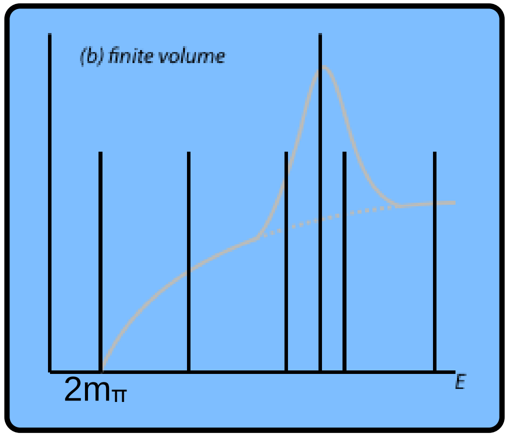
Spectrum of finite volume field theory

Missing states: "continuum" of multi-particle scattering states

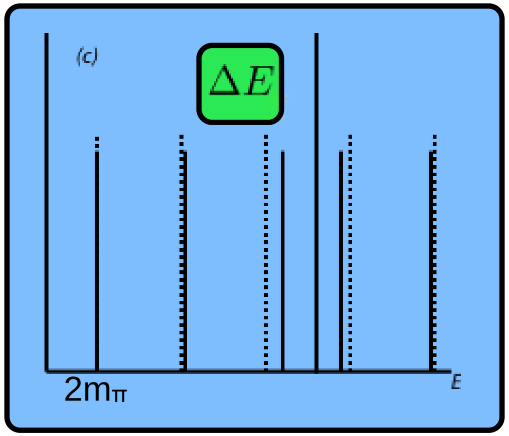


Infinite volume: continuous spectrum

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$



Finite volume: discrete spectrum



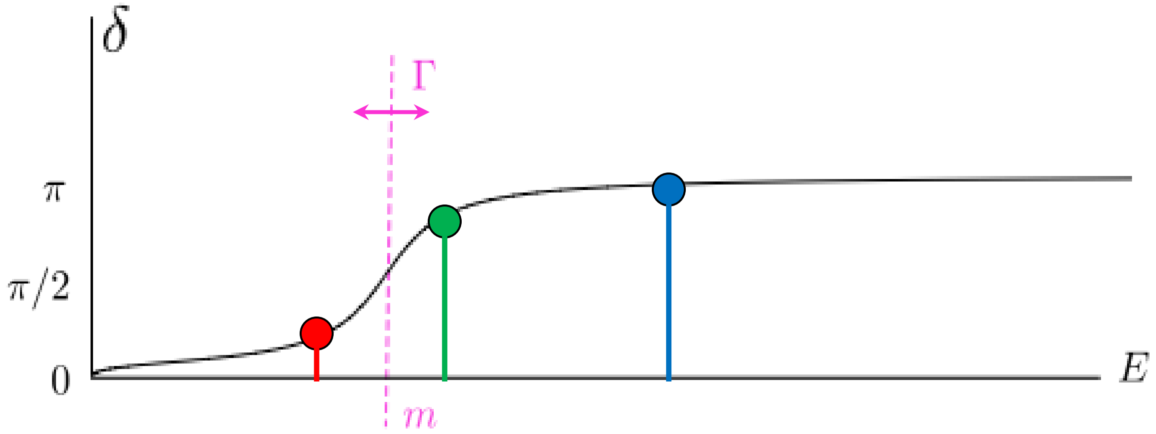
Deviation from (discrete) free energies depends upon interaction - contains information about scattering phase shift

ΔE(L) ↔ δ(E) : Lüscher method

Finite volume scattering

Lüscher method
-scattering in a periodic cubic box (length **L**)
-finite volume energy levels **E(L)** $\rightarrow$ **δ(E)**

E.g. just a single elastic resonance



e.g.

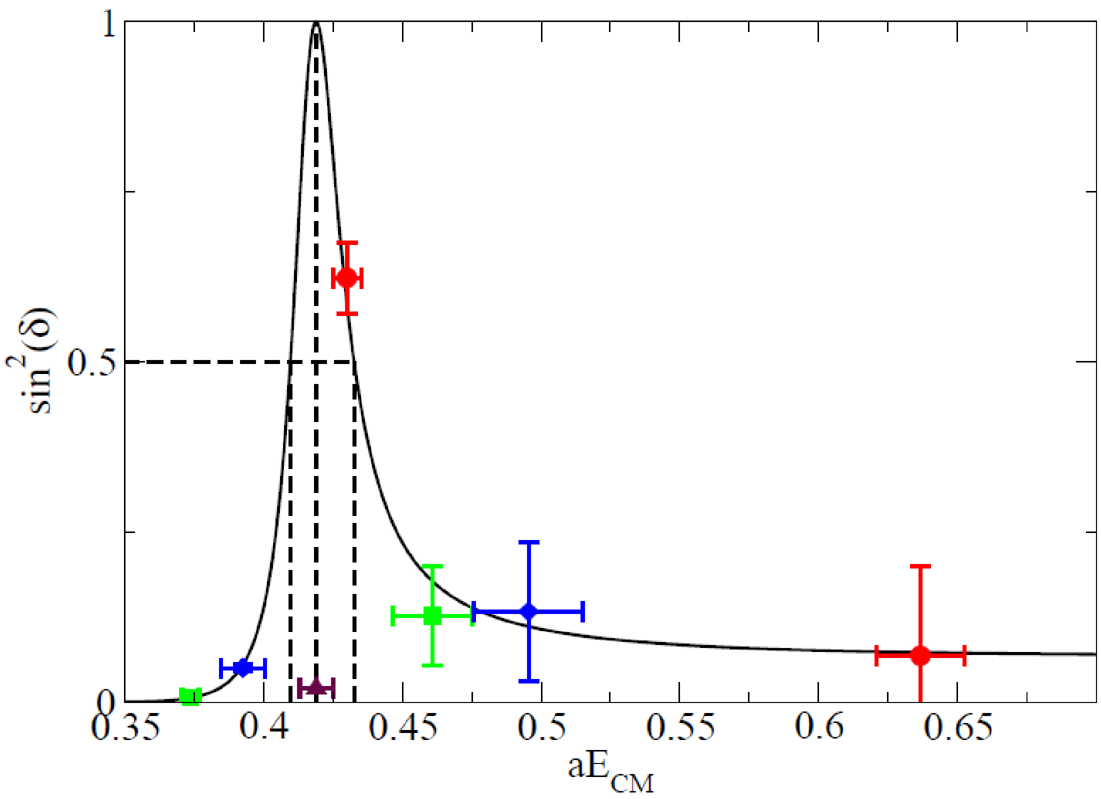
$$\pi\pi \rightarrow \rho \rightarrow \pi\pi$$

$$\pi N \rightarrow \Delta \rightarrow \pi N$$

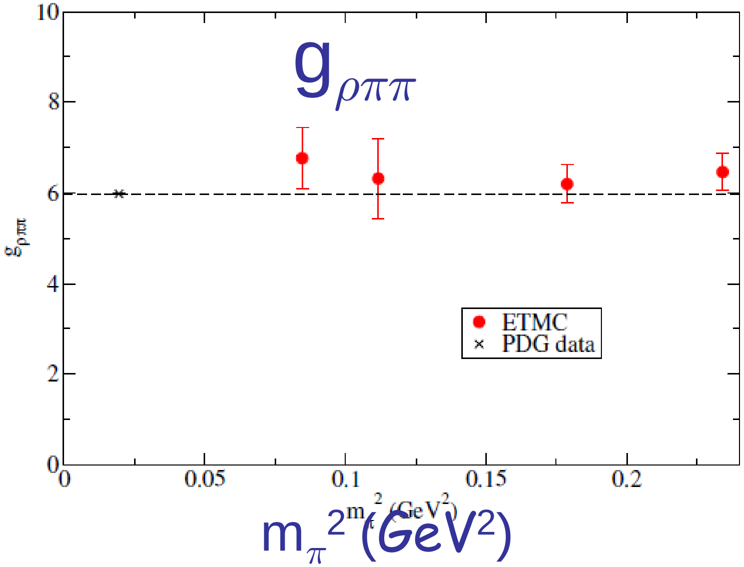
At some **L ,** have discrete excited energies

I=1 $\pi\pi$: the "ρ"

Extract $\delta_1(E)$ at discrete E



Extracted coupling: stable in pion mass



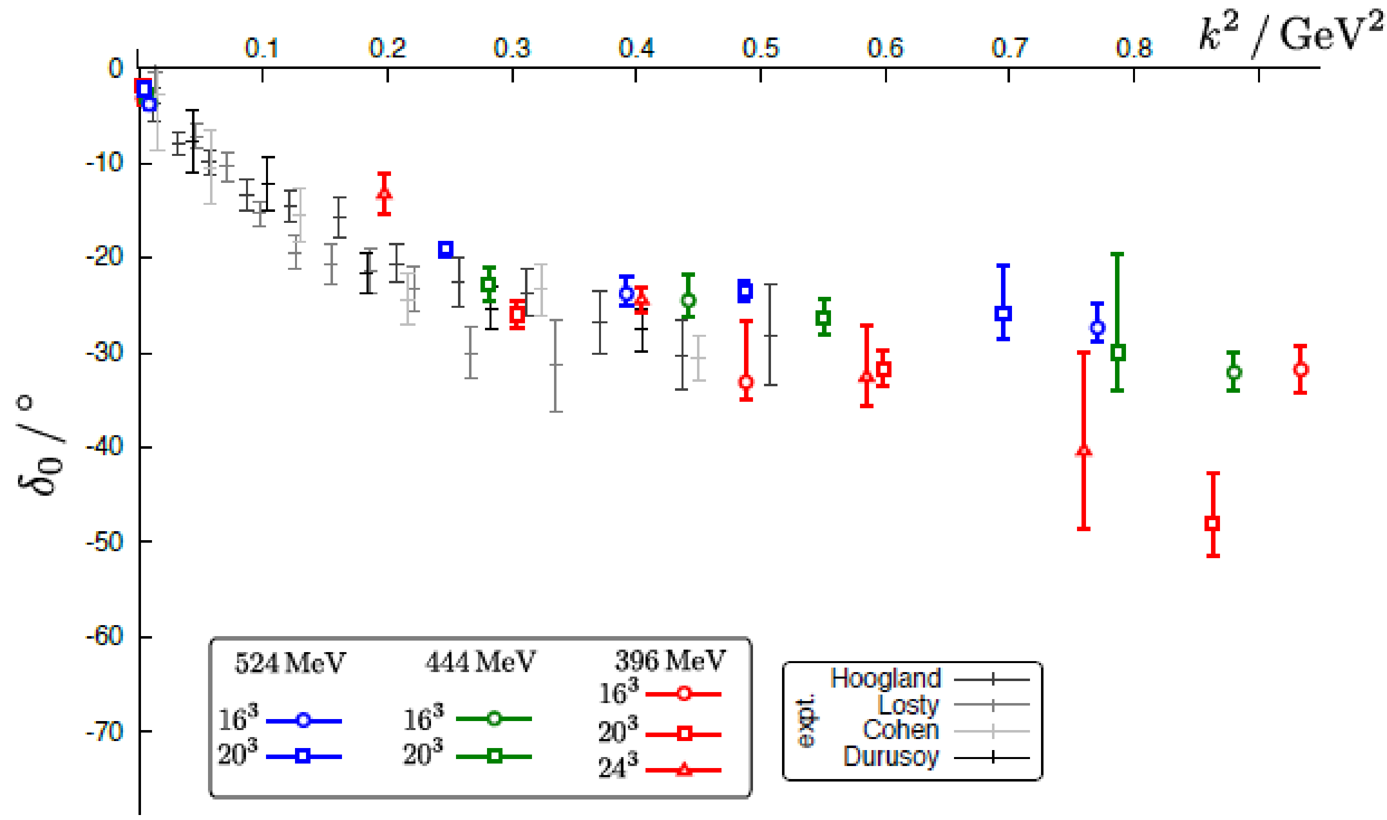
Stability a generic feature of couplings??

Feng, Jansen, Renner, 1011.5288

Phase Shifts demonstration: I=2 $\pi\pi$

$\pi\pi$ isospin=2

Extract $\delta_0(E)$ at discrete E



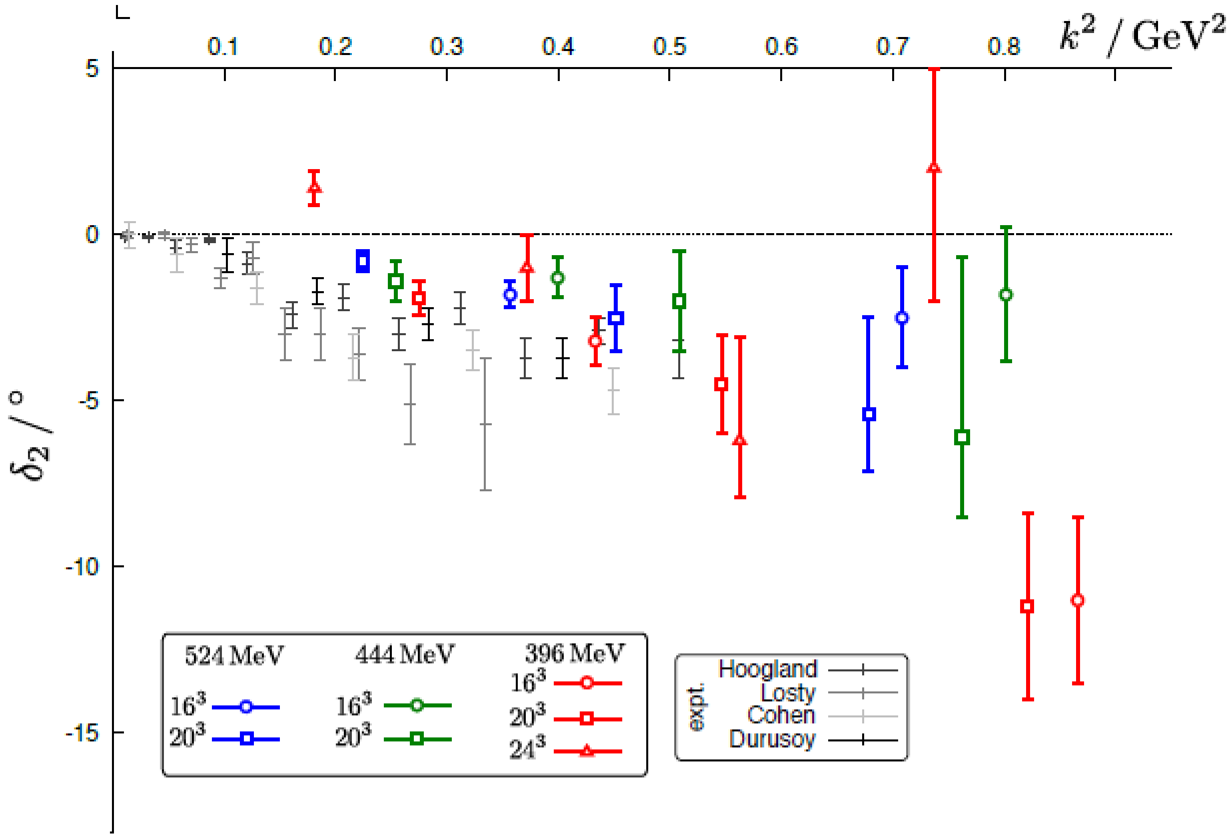
No discernible pion mass dependence

1011.6352 (PRD)

Phase Shifts: demonstration

$\pi\pi$ isospin=2

$\delta_2(E)$



Form Factors

What is a form-factor off of a resonance?

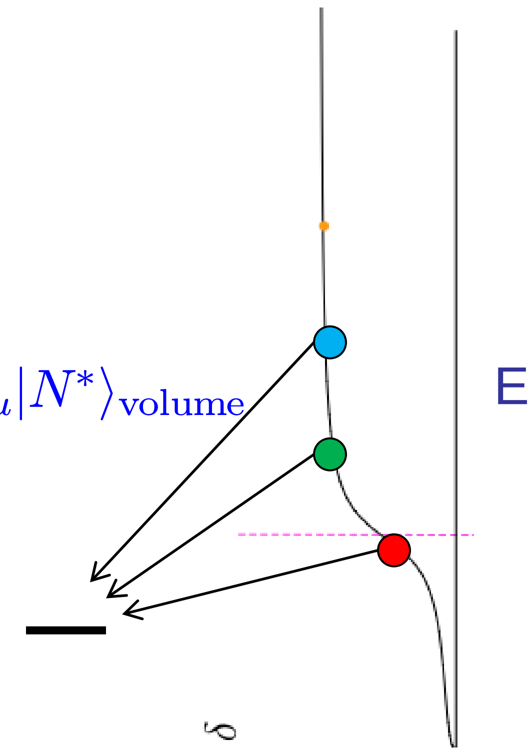
What is a resonance? Spectrum first!

Extension of scattering techniques:
- Finite volume matrix element modified

$$\langle N|J_\mu|N^*\rangle_\infty(Q^2,E) \leftarrow [\delta'(E)+\Phi'(E)]\,\langle N|J_\mu|N^*\rangle_{\mathrm{volume}}$$

Phase shift

Kinematic factor



Requires excited level transition FF's: some experience
- Charmonium E&M transition FF's (1004.4930)
- Nucleon 1st attempt: "Roper"->N (0803.3020)

Range: few GeV^2
Limitation: spatial lattice spacing

(Very) Large Q^2

Standard requirements:

$$\frac{1}{L} \ll m_\pi, m_N, Q \ll \frac{1}{a}$$

Cutoff effects: lattice spacing $(a_s)^{-1} \sim 1.6$ GeV

Appeal to renormalization group: *Finite-Size* scaling

Use short-distance quantity: compute perturbatively and/or parameterize

$$R(Q^2) = \frac{F(s^2 Q^2)}{F(Q^2)}, \qquad s = 2$$

"Unfold" ratio only at low Q^2 / s^{2N}

$$F(Q^2) = R(Q^2/s^2) R(Q^2/s^4) \cdots R(Q^2)/s^{2N})\ F(Q^2/s^{2N})$$

For $Q^2 = 100$ GeV^2 and N=3, $Q^2 / s^{2N} \sim 1.5$ GeV^2

Initial applications: factorization in pion-FF

D. Renner

Hadronic Decays

Some candidates: determine phase shift
Somewhat elastic

$m_\pi \sim 400$ MeV